CS 410/510: Advanced Programming

Abstract Datatypes + Functions as Data

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Back to Builders

Building Builders:

```
data Builder a = Builder { build::Int -> (a, [NFATrans], Int) }

newState :: Builder NFAState
newState  = Builder (\n -> (n, [], n+1))

addTrans :: NFATrans -> Builder ()
addTrans t = Builder (\n -> ((), [t], n))

returnB  :: a -> Builder a
returnB x = Builder (\n -> (x, [], n))

bindB    :: Builder a -> (a -> Builder b) -> Builder b
bindB b f = Builder (\n -> let (x, ts1, n1) = build b n
                               (y, ts2, n2) = build (f x) n1
                           in  (y, ts1++ts2, n2))

instance Monad Builder where
  return = returnB
  (>>=)  = bindB
```

These are the only operations that we will use to build Builders …

Example:

Example:

```
nfab' (C c) f  = do s <- newState
                    addTrans (Transition (Char c) s f)
                    return s
```

is syntactic sugar for:

```
nfab' (C c) f  = newState >>= \s ->
                 addTrans (Transition (Char c) s f) >>= \_ ->
                 return s
```

which, in turn, is an abbreviation for:

```
nfab' (C c) f  = newState `bindB` \s ->
                 addTrans (Transition (Char c) s f) `bindB` \_ ->
                 returnB s
```

Under the Hood:

Let's break this down:

```
nfab' (C c) f  = newState `bindB` \s ->
                 addTrans (Transition (Char c) s f) `bindB` \_ ->
                 returnB s
```

becomes:

```
nfab' (C c) f  = newState `bindB` g
 where
  g s = addTrans (Transition (Char c) s f) `bindB` h
  h _ = returnB s
```

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becomes:

```
nfab' (C c) f  = Builder (\n->(n, [Transition (Char c) n f], n+1))
```

For example:

```
build (nfab' (C 'a') 0) 1  = (1, [Transition (Char 'a') 1 0], 2))
```

Back to Building Builders:

```
data Builder a = Builder { build::Int -> (a, [NFATrans], Int) }

newState :: Builder NFAState
newState  = Builder (\n -> (n, [], n+1))

addTrans :: NFATrans -> Builder ()
addTrans t = Builder (\n -> ((), [t], n))

returnB  :: a -> Builder a
returnB x = Builder (\n -> (x, [], n))

bindB     :: Builder a -> (a -> Builder b) -> Builder b
bindB b f = Builder (\n -> let (x, ts1, n1) = build b n
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instance Monad Builder where
  return = returnB
  (>>=)  = bindB
```

These are the only operations that we will use to build Builders …

Bad Builders:

We don’t want programmers to start creating arbitrary builders, because they might accidentally (or intentionally) break the invariants that we have for our Builder structures:

```
bad = Builder (\n -> (n, [epsilon n (n-1)], n-2))
```

Back to Building Builders:

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newState :: Builder NFAState
newState  = Builder (\n -> (n, [], n+1))

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addTrans t = Builder (\n -> ((), [t], n))

returnB  :: a -> Builder a
returnB x = Builder (\n -> (x, [], n))

bindB    :: Builder a -> (a -> Builder b) -> Builder b
bindB b f = Builder (\n -> let (x, ts1, n1) = build b n
                               (y, ts2, n2) = build (f x) n1
                           in  (y, ts1++ts2, n2))

instance Monad Builder where
  return = returnB
  (>>=)  = bindB
```

These are the only operations that we **can** use to build Builders …

Using a Haskell Module:

```
module Builder(Builder, build, newState, addTrans) where

data Builder a

build    :: Builder a -> Int -> (a, [NFATrans], Int) }

newState :: Builder NFAState

addTrans :: NFATrans -> Builder ()

instance Monad Builder where
  return = returnB
  (>>=)  = bindB
```

Inside the module: the full implementation of the Builder type is visible

Outside the module: only the names and types of the Builder type and operations are visible

Why we used data …

- Did you wonder why I'd used:

  ```
  data Builder a = Builder (Int -> (a, [NFATrans], Int))
  ```

  instead of just defining:

  ```
  type Builder a = Int -> (a, [NFATrans], Int)
  ```

  ?

- We could make the original code work just as well if we eliminated every use of the build function and the Builder constructor function

- But using a datatype makes it possible to distinguish Builder values from other functions that happen to have the same type … and makes it possible to conceal that implementation in a module

Monads:

- Monads are abstract types that represent computations
- Every monad has at least at return and a bind (>>=) operation
- If M is a monad, then a value of type M T represents:
  - A computation that returns values of type T
  - That uses the special features of monad M

The IO Monad

The IO Type:

- The type IO t represents interactive programs that produce values of type t
- The main function in every Haskell program is expected to have type IO ()
- If you write an expression of type IO t at the Hugs prompt, it will be evaluated as a program and the result discarded
- If you write an expression of some other type at the Hugs prompt, it will be turned in to an IO program using:

```
print :: (Show a) => a -> IO ()
print = putStrLn . show
```

I/O Primitives:

- putChar c is a program that prints the single character c on the console:

```
putChar :: Char -> IO ()
```

- (>>) is an infix operator that glues two IO programs together, returning the result of the second

```
(>>) :: IO a -> IO b -> IO b
```

- For example: putChar 'h' >> putChar 'i'

putStr and putStrLn:

- Now, for example, we can define:

```
putStr          :: String -> IO ()
putStr          = foldr1 (>>) . map putChar

putStrLn        :: String -> IO ()
putStrLn s      = putStr s >> putChar "\n"
```

- Alternatively

```
putStr          = mapM_ putChar
```

using the primitives

```
mapM            :: (a -> IO b) -> [a] -> IO [b]
mapM_           :: (a -> IO b) -> [a] -> IO ()
```

“do-notation”:

- Syntactic sugar for writing (monadic) IO programs:

  do p_1
  p_2
  …
  p_n

  is equivalent to:

  p_1 >> p_2 >> … >> p_n

  and can also be written:

  do p_1; p_2; …; p_n or **do** { p_1; p_2; …; p_n }

return:

- We can make a program that returns x without doing any I/O using return x:

  ```
  return :: a -> IO a
  ```

- Note that return is not quite like the return we know from imperative languages:

  (**do** return 1; return 2) = return 2

Using Return Values:

- How can we use returned values?
- Another important primitive:

  ```
  (>>=) :: IO a -> (a -> IO b) -> IO b
  ```

- For example, putChar 'a' is equivalent to:

  ```
  return 'a' >>= putChar :: IO ()
  ```

- In fact, return and (>>=) satisfy laws:

  ```
  return e >>= f      =  f e
  p >>= return =  p
  ```

Relating >>= and >>:

- (>>) can be defined as a special form of (>>=) that ignores the result of the first program:

```
p >> q        = p >>= (\ _ -> q)
```

- Special laws:

```
(p>>q) >> r  =  p >> (q >> r)
(p >>= f) >>= g
     = p >>= (\x -> f x >>= g)
```

Defining mapM and mapM_:

```
mapM_              :: (a -> IO b) -> [a] -> IO ()
mapM_ f []         = return ()
mapM_ f (x:xs)     = f x               >>
                     mapM_ f xs

mapM               :: (a->IO b) -> [a]->IO [b]
mapM f []          = return []
mapM f (x:xs)      = f x            >>= \y ->
                     mapM f xs  >>= \ys->
                     return (y:ys)
```

Extending "do-notation":

We can bind the results produced by IO programs to variables using an extended form of do-notation. For example:

```
do  x1 <- p1
    ...
    xn <- pn
    q
```

all "generators" should have the same indentation

last item must be an expression

is equivalent to:

```
p1 >>= \x1 ->
...
pn >>= \xn ->
q
```

variables introduced in a generator are in scope for the rest of the expression

The "v <-" portion of a generator is optional and defaults to "_ <-" if

Defining mapM and mapM_:

```
mapM_               :: (a -> IO b) -> [a] -> IO ()
mapM_ f []          = return ()
mapM_ f (x:xs)      = do  f x
                          mapM_ f xs

mapM                :: (a->IO b) -> [a]->IO [b]
mapM f []           = return []
mapM f (x:xs)       = do  y  <- f x
                          ys <- mapM f xs
                          return (y:ys)
```

getChar:

- A simple primitive for reading a single character:

```
getChar :: IO Char
```

- A simple example:

```
echo :: IO a
echo = do c <- getChar
          putChar c
          echo
```

Reading a Complete Line:

```
getLine  :: IO String
getLine  = do c <- getChar
              if c=='\n'
                then return ""
                else do cs <- getLine
                        return (c:cs)
```

Alternative:

```
getLine   :: IO String
getLine   = loop []

loop      :: String -> IO String
loop cs   =  do c <- getChar
               case c of
                 '\n' -> return (reverse cs)
                 '\b' -> case cs of
                           []     -> loop cs
                           (c:cs) -> loop cs
                 c    -> loop (c:cs)
```

Simple File I/O:

- Read contents of a text file:

```
readFile :: FilePath -> IO String
```

- Write a text file:

```
writeFile :: FilePath -> String -> IO ()
```

- Example: Number lines

```
numLines inp out
    = do s <- readFile inp
         writeFile out (unlines (f (lines s)))
f = zipWith (\n s -> show n ++ s) [1..]
```

Handle-based I/O:

```
import IO

stdin, stderr, stdout :: Handle

openFile     :: FilePath -> IOMode -> IO Handle

hGetChar     :: Handle -> IO Char

hPutChar     :: Handle -> Char -> IO ()

hClose       :: Handle -> IO ()
```

References:

```
import Data.IORef

data IORef a = ...

newIORef  :: a -> IO (IORef a)

readIORef :: IORef a -> IO a

writeIORef :: IORef a -> a -> IO ()
```

IO as an Abstract Type:

- IO is a primitive type constructor in Haskell with a large but limited set of operations:

```
return    :: a -> IO a
(>>=)     :: IO a -> (a -> IO b) -> IO b

putChar        :: Char -> IO ()
getChar        :: IO Char
…
```

There is No Escape from IO!

- There are plenty of ways to construct expressions of type IO t
- Once a program is "tainted" with IO, there is no way to "shake it off"
- There is no primitive of type IO t -> t that runs a program and returns its result
- Only two ways to run an IO program:
  - Setting it as your main function in GHC
  - Typing it at the prompt in Hugs/GHCi

Functions as Data

Functions as Data:

- Obviously, functions are an important tool that we use to manipulate data in functional programs

- But functions are first-class values in their own right, so they can also be used as data …

Sets as Functions:

```
type Set a          = a -> Bool

isElem              :: a -> Set a -> Bool
x `isElem` s        = s x

univ                :: Set a
univ                = \x -> True

empty               :: Set a
empty               = \x -> False

singleton           :: Eq a => a -> Set a
singleton v         = \x -> (x==v)
```

… continued:

```
(\/)      :: Set a -> Set a -> Set a
s \/ t    = \x -> s x || t x

(/\)      :: Set a -> Set a -> Set a
s /\ t    = \x -> s x && t x
```

- Stylistic detail: I write `op x y = \z -> …` to emphasize that `op` is a binary operator that returns a function as its result.
- Equivalent to: `op x y z = …`

Other Operations?

- Can I enumerate the elements of a Set?

```
toList   :: Set a -> [a]
```

- Can I compare sets for equality?

```
setEq :: Set a -> Set a -> Bool
```

- Can I test for subsets?

```
subset :: Set a -> Set a -> Bool
```

The Data Alternative:

```
data Set a = Empty
           | Univ
           | Singleton a
           | Union (Set a) (Set a)
           | Intersect (Set a) (Set a)
```

Now we can implement empty, univ, singleton, (\/) and (/\) directly in terms of these constructors: For example:

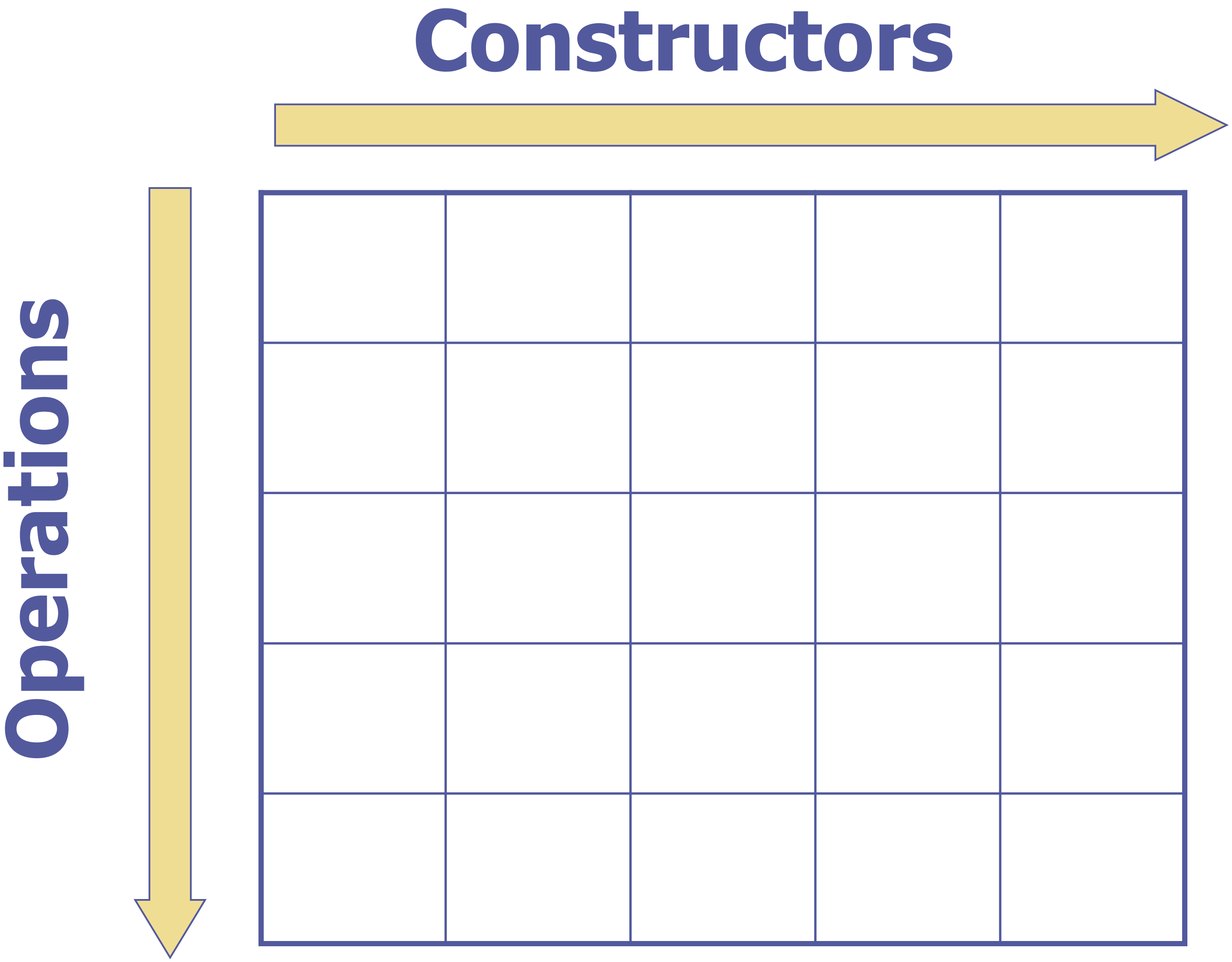
```
empty = Empty
```

Testing for Membership:

```
isElem :: Eq a => a -> Set a -> Bool
x `isElem` Empty              = False
x `isElem` Univ               = True
x `isElem` Singleton y        = (x==y)
x `isElem` Union l r          = x `isElem` l
                             || x `isElem` r
x `isElem` Intersect l r      = x `isElem` l
                             && x `isElem` r
```

Same code, different distribution …

Rows and Columns:



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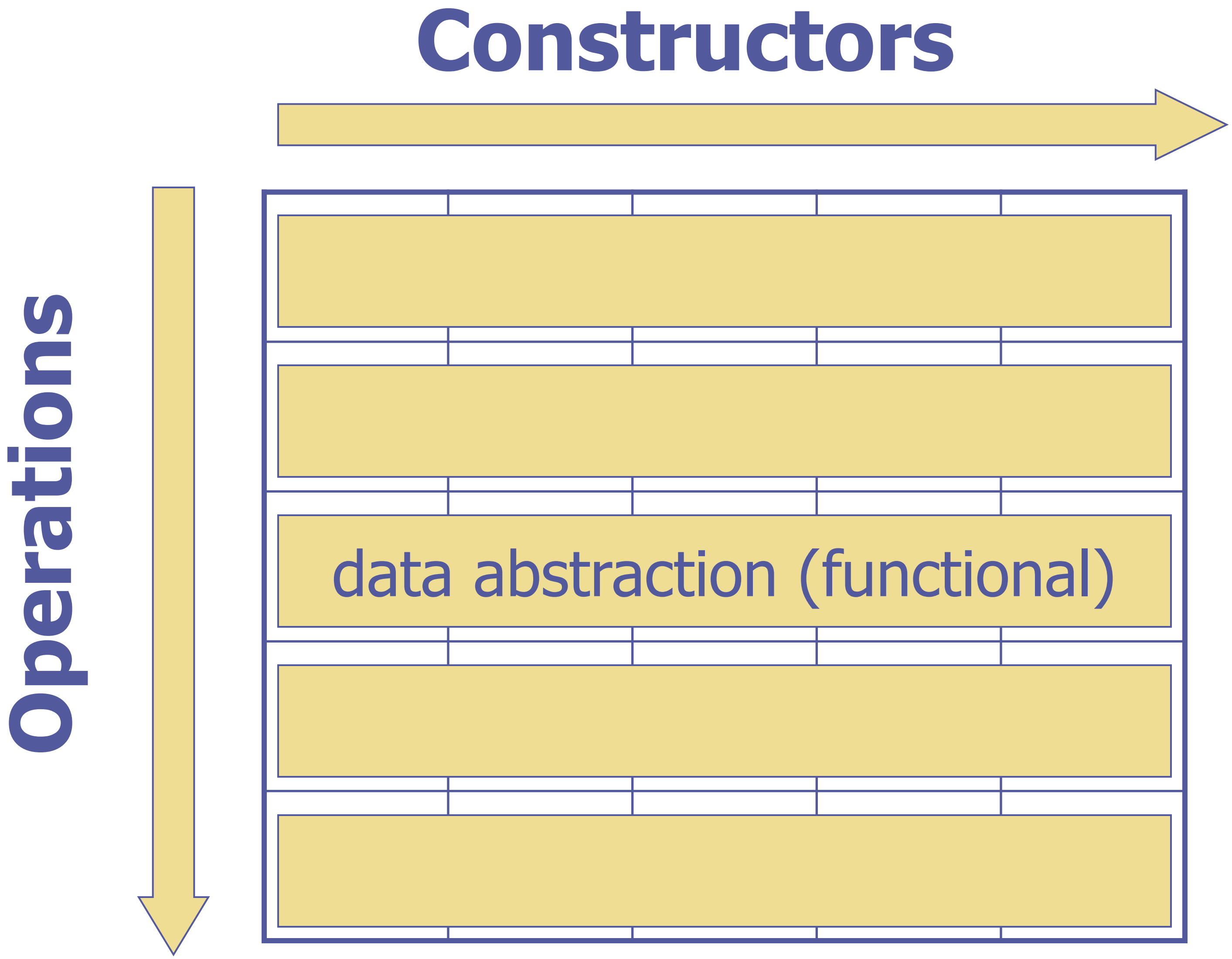

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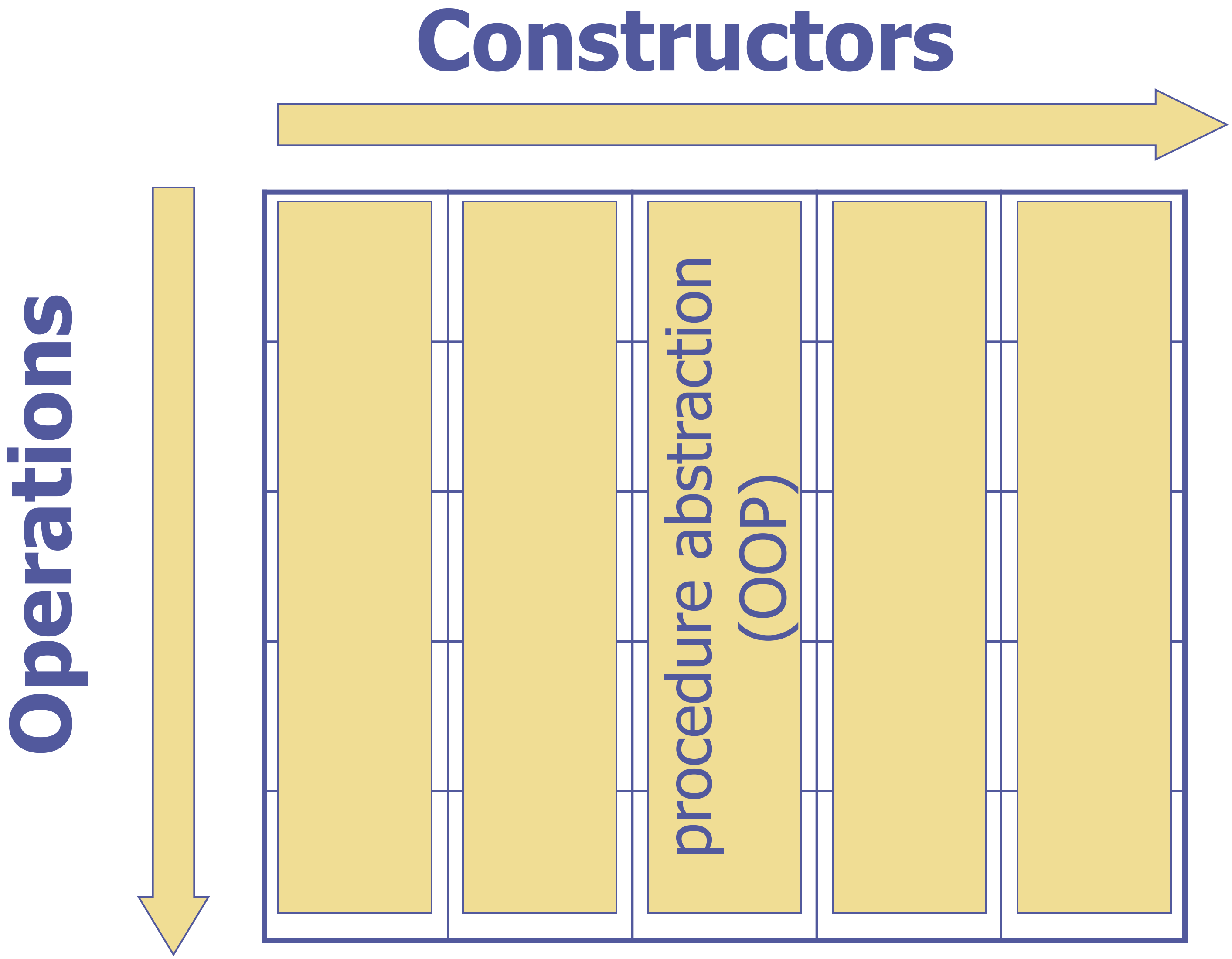

… continued:

Representing sets using functions:

- “Easy” to add new constructors
- “Hard” to add new operations

Representing sets using trees:

- “Easy” to add new operations
- “Hard” to add new constructors

- Can we make it “easy” in both dimensions?
- A classic challenge for extensible software

Parser Combinators

Parsers:

```
data Parser a
        = Parser { applyP :: String -> [(a, String)]}

applyP      :: Parser a -> String -> [(a, String)]

noparse     :: Parser a
noparse     = Parser (\s -> [])

ok          :: a -> Parser a
ok x        = Parser (\s -> [(x, s)])
```

Parsers as a Monad:

```
instance Monad Parser where
  return x = ok x
  p >>= f = Parser (\s ->
                    [ (y,s2) | (x,s1) <- applyP p s,
                               (y,s2) <- applyP (f x) s1 ])

(***)   :: Parser a -> (a -> b) -> Parser b
p *** f = do x <- p
             return (f x)
```

… continued:

```
item   :: Parser Char
item   = Parser (\s -> case s of
                            []     -> []
                            (t:ts) -> [(t,ts)])

sat     :: (Char -> Bool) -> Parser Char
sat p   = Parser (filter (p . fst) . applyP item)

is      :: Char -> Parser Char
is c    = sat (c==)
```

Examples:

```
digit     :: Parser Int
digit     = sat isDigit >>= \d -> ord d – ord '0'

alpha, lower, upper :: Parser Char
alpha   = sat isAlpha
lower   = sat isLower
upper   = sat isUpper

string          :: String -> Parser String
string ""       = return ""
string (c:cs) = do char c; string cs; return (c:cs)
```

Alternatives:

```
infixr 4 |||

(|||)     :: Parser a -> Parser a -> Parser a
p ||| q   = \s -> p s ++ q s

ex2       :: Parser Char
ex2       = alpha ||| ok '0'
```

Sequences:

```
infixr 6 >>>

(>>>)    :: Parser a -> Parser b -> Parser (a,b)
p >>> q = do x <- p; y <- q; return (x,y)

ex3 :: Parser (Char, Char)
ex3 = sat isDigit >>> sat (not . isDigit)
```

Repetition:

```
many      :: Parser a -> Parser [a]
many p    = many1 p ||| return []

many1     :: Parser a -> Parser [a]
many1 p   = do x <- p
               xs <- many p
               return (x:xs)
```

“Lexical Analysis”:

```
number :: Parser Int
number = many1 digit
              *** foldl1 (\a x -> 10*a+x)
```

Context-Free Parsing:

Consider the following grammar:

```
expr   = term "+" expr
       | term "-" expr
       | term
term   = atom "*" term
       | atom "/" term
       | atom
atom   = "-" atom
       | "(" expr ")"
       | number
```

Context-Free Parsing:

A little refactoring:

```
expr    = term ("+" expr | "-" expr | ε)
term    = atom ("*" term | atom "/" | ε)
atom    = "-" atom
          | "(" expr ")"
          | number
```

Context-Free Parsing:

Translation into Haskell:

```
expr, term, atom :: Parser Int

expr    = term >>= \x ->
            (string "+" >> expr >>>= \y -> ok (x+y)) |||
            (string "-" >> expr >>>= \y -> ok (x-y))  |||
            ok x
```

… continued:

```
term
    = atom >>= \x ->
        (string "*" >> term >>= \y -> ok (x*y))     |||
        (string "/" >> term >>= \y -> ok (x`div`y)) |||
        ok x

atom
    = (string"-" >> atom) *** negate
        |||
      (string "(" >> expr >>= \n -> string ")" >> ok n)
        |||
      number
```

Examples:

```
Main> expr "1+2*3"
[(7,""),(3,"*3"),(1,"+2*3")]

Main> expr "(1+2)*3"
[(9,""),(3,"*3")]

Main> expr "---------1+2*----3"
[(5,""),(1,"*----3"),
  (-1,"+2*----3")]
```

Introducing a Helper:

```
Parse          :: Parser a -> String -> [a]
parse p s      = [ x | (x,"") <- applyP p s ]
```

```
Main> parse expr "1+2*3"
[7]
Main> parse expr "(1+2)*3"
[9]
Main> parse expr "---------1+2*----3"
[5]
Main>
```

Declarative Programming:

- Although it may not be immediately apparent, the structure of our program directly mimics the structure of the problem (i.e., the grammar)
- In principal, we get to express our parser at a high-level, and we don't have to worry about the details of how it is implemented
- In practice, we do (left recursion, exponential behavior, space leaks, etc..)

Constructing Abstract Syntax:

- Suppose that we define a datatype to represent arithmetic expressions:

```
data Expr = Add Expr Expr
          | Sub Expr Expr
          | Mul Expr Expr
          | Div Expr Expr
          | Neg Expr
          | Num Int
            deriving Show
```

- How can I construct an Expr from an input string?

... continued:

```
absyn :: Parser Char Expr
absyn  = expr
 where
  expr    = term >>= \x ->
              (string "+" >> expr >>= \y -> ok (Add x y)) |||
              (string "-" >> expr >>= \y -> ok (Sub x y)) |||
              ok x

  term    = atom >>= \x ->
              (string "*" >> term >>= \y -> ok (Mul x y)) |||
              (string "/" >> term >>= \y -> ok (Div x y)) |||
              ok x

  atom    =  (string "-" >> atom *** Neg)
            |||
             (string "(" >> expr >>= \n -> string ")" >> ok n)
            |||
             (number *** Num)
```

Examples:

```
Main> parse absyn "1+2*3"
[Add (Num 1) (Mul (Num 2) (Num 3))]

Main> parse absyn "------1"
[Neg (Neg (Neg (Neg (Neg (Neg (Num 1))))))]

Main> parse expr  "------1"
[1]

Main>
```

Context-Sensitive Parsing:

We can easily go beyond context-free parsing in this framework:

```
brack   :: Parser String
brack   = do c <- char
             xs <- many (sat (c/=))
             sat (c==)
             return xs
```

Summary:

- Powerful ideas!
- Abstract types
- Monads as abstract types for computations
- Using functions as data
- Parser combinators